

Study Guide on LDF Curve-Fitting and Stochastic Reserving for the Society of Actuaries (SOA) Exam GIADV: Advanced Topics in General Insurance

(Based on David R. Clark's Paper "[LDF Curve-Fitting and Stochastic Reserving](#)")
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Problem LDFCF-1. What are two main reasons why there is a range of possible outcomes around the "expected" reserve? (Clark, p. 3)

Solution LDFCF-1. The following are the two main reasons why there is a range of possible outcomes around the "expected" reserve (Clark, p. 3):

Reason 1. Random "process" variance

Reason 2. Uncertainty in the estimate of the expected value

Problem LDFCF-2. What are two key elements of a statistical loss-reserving model? (Clark, p. 3)

Solution LDFCF-2. The following are the two key elements of a statistical loss-reserving model (Clark, p. 3):

Element 1. The expected amount of loss to emerge in some time period

Element 2. The distribution of actual emergence around the expected value

Problem LDFCF-3. The model developed by Clark estimates the expected amount of loss based on *which* two estimates? (Clark, p. 5)

Solution LDFCF-3. The model developed by Clark estimates the expected amount of loss based on:

(i) An estimate of the ultimate loss by year;

(ii) An estimate of the pattern of loss emergence. (Clark, p. 5)

Problem LDFCF-4.

(a) Let LDF_x be the loss-development factor at time x . What is the formula for the cumulative distribution function (CDF) $G(x)$, which represents the cumulative percent of losses reported or paid (depending on whether reported losses or paid losses are being evaluated) as of time x ?

(b) What specific assumption does Clark make regarding the time index x ?

(Clark, p. 5)

Solution LDFCF-4.

(a) $G(x) = 1/LDF_x$.

(b) Clark assumes that the time index x represents the time from the “average” accident date to the evaluation date. (Clark, p. 5)

Problem LDFCF-5.

(a) What are the conceptual descriptions of the parameters θ and ω , common to the Weibull and Loglogistic distributions? (Clark, p. 5)

(b) What is another name for the Loglogistic curve? (Clark, pp. 5-6)

(c) Which distribution, the Weibull or the Loglogistic, generally provides a smaller “tail” factor? (Clark, p. 6)

Solution LDFCF-5.

(a) θ is the **scale parameter**.

ω is the **shape** or **warp parameter**.

(b) Another name for the Loglogistic curve is the **inverse power** curve.

(c) The **Weibull distribution** generally provides a smaller “tail” factor than the Loglogistic distribution. (Clark, pp. 5-6)

Problem LDFCF-6.

(a) What is the formula for $G(x \mid \omega, \theta)$, the cumulative distribution function of the Loglogistic distribution?

(b) What is the formula for LDF_x if x follows a Loglogistic distribution?

(c) What is the formula for $G(x \mid \omega, \theta)$, the cumulative distribution function of the Weibull distribution?

(d) What assumption does the use of both the Loglogistic and Weibull curves make, and what is a situation that, if expected, should preclude using them? (Clark, p. 6)

Solution LDFCF-6.

(a) Loglogistic distribution CDF: $G(x \mid \omega, \theta) = x^\omega / (x^\omega + \theta^\omega)$.

(b) Loglogistic distribution LDF: $LDF_x = 1 + \theta^\omega / x^\omega$.

(c) Weibull distribution CDF: $G(x \mid \omega, \theta) = 1 - \exp(-(x/\theta)^\omega)$.

(d) Both the Loglogistic and Weibull curves assume that **expected loss emergence follows a strictly increasing pattern**. If there is *expected* negative development, these curves should not be used. (Actual negative development can be accommodated to a certain extent, as long as it is minor and the overall expected development is positive.) (Clark, p. 6)

Problem LDFCF-7. What are three advantages to using parametrized curves to describe the expected loss-emergence pattern? (Clark, p. 6)

Solution LDFCF-7.

Advantage 1. The estimation problem is simplified, because we only need to estimate the two parameters.

Advantage 2. We can use data that are not strictly from a triangle with evenly spaced evaluation dates.

Advantage 3. The final indicated pattern is a smooth curve and does not follow every random movement in the historical age-to-age factors.

Problem LDFCF-8. What are the key differences between the LDF method and the Cape Cod method in the assumptions regarding *independence* of ultimate losses in each accident year? (Clark, pp. 6-7)

Solution LDFCF-8. The **LDF method** assumes that the ultimate loss amount in each accident year is independent of the losses in other years.

The **Cape Cod method** assumes that there is a known relationship between the amount of ultimate loss expected in each of the years of the historical period, and that this relationship is identified by an exposure base (usually on-level premium, but possibly another index such as sales or payroll) which can be assumed to be proportional to expected loss. (Clark, pp. 6-7)

Problem LDFCF-9. Let $\mu_{AY:x,y}$ be the expected incremental loss amount in accident year AY between ages x and y.

(a) For the Cape Cod method, given premium P_{AY} for accident year AY, expected loss ratio ELR, and a cumulative distribution function $G(t \mid \omega, \theta)$ for any nonnegative values of t, what is the expression for $\mu_{AY:x,y}$?

(b) How many parameters are involved in this application of the Cape Cod method, and what are they? (Clark, p. 7)

Solution LDFCF-9.

(a) $\mu_{AY:x,y} = P_{AY} * ELR * [G(y \mid \omega, \theta) - G(x \mid \omega, \theta)]$.

(b) **3 parameters: ELR, ω , θ**

(Clark, p. 7)

Problem LDFCF-10. Let $\mu_{AY:x,y}$ be the expected incremental loss amount in accident year AY between ages x and y.

(a) For the LDF method, given ultimate losses ULT_{AY} for accident year AY and a cumulative distribution function $G(t \mid \omega, \theta)$ for any nonnegative values of t, what is the expression for $\mu_{AY:x,y}$?

(b) How many parameters are involved in this application of the LDF method, and what are they? (Clark, p. 7)

Solution LDFCF-10.

(a) $\mu_{AY:x,y} = ULT_{AY} * [G(y \mid \omega, \theta) - G(x \mid \omega, \theta)]$.

(b) **(n+2) parameters: n accident years, ω , θ**

(Clark, p. 7)

Problem LDFCF-11. What problem does the use of Clark's LDF method pose, that Clark's Cape Cod method can overcome? (Clark, pp. 7-8)

Solution LDFCF-11. Clark’s LDF method suffers from the problem of **overparametrization**, where there is a separate parameter for each accident year. With a loss-development triangle, where there are relatively few data points, having too many parameters is a serious issue. The Cape Cod method overcomes this problem by only utilizing three parameters, the expected loss ratio and the two parameters of the parametrized curve. (Clark, pp. 7-8)

Problem LDFCF-12. It is assumed that loss emergence (using a time scale of years) follows a Weibull distribution with parameters $\theta = 4$ and $\omega = 2$. Moreover, it is given that, for Accident Year 3030, the expected loss ratio is 46%, and the premium is 3,000,000.

Using Clark’s Cape Cod method, what is the expected incremental loss amount for Accident Year 3030 between 1 year and 4 years?

Solution LDFCF-12.

We use the formula $\mu_{AY:x,y} = P_{AY} * ELR * [G(y \mid \omega, \theta) - G(x \mid \omega, \theta)]$. We are asked to find $\mu_{AY:1,4}$, where $P_{AY} = 3,000,000$, and $ELR = 46\%$.

For a Weibull distribution, $G(t \mid \omega, \theta) = 1 - \exp(-(t/\theta)^\omega)$.

We find $G(1 \mid 2, 4) = 1 - \exp(-(1/4)^2) = 0.0605869372$.

We find $G(4 \mid 2, 4) = 1 - \exp(-(4/4)^2) = 0.6321205588$.

Thus, $\mu_{AY:1,4} = 3000000 * 0.46 * (0.6321205588 - 0.0605869372) = \mathbf{788,716.4}$.

Problem LDFCF-13. Fill in the blanks (Clark, p. 8): Compared to the LDF method, the Cape Cod method may have higher _____, but will usually produce significantly smaller _____. This is due to the value of the _____ in the _____ provided by the user.

Solution LDFCF-13. Compared to the LDF method, the Cape Cod method may have higher **process variance**, but will usually produce significantly smaller **estimation error**. This is due to the value of the **information** in the **exposure base** provided by the user. (Clark, p. 8)

Problem LDFCF-14. Identify and briefly describe the two “pieces” of variance that Clark’s model estimates. (Clark, p. 9)

Solution LDFCF-14. The two “pieces” of variance are **(i) process variance** – the “random” amount of the variance and **(ii) parameter variance** – the uncertainty in the estimator. (Clark, p. 9)

Problem LDFCF-15. Let $\mu_{AY,t}$ be the *expected* incremental loss amount in accident year AY over time t. Let $c_{AY,t}$ be the *actual* incremental loss amount in accident year AY over time t. Let p be the number of parameters and n be the number of observations.

(a) What is the formula for the approximation for σ^2 , which in this case is equal to the ratio of the variance over the mean of the loss emergence?

(b) The expression in part (a) is equivalent to what kind of term known in statistical theory?

(c) For estimating the parameters in Clark’s model, it is assumed that the actual incremental loss emergence c follows *what* distribution? (Clark, p. 9)

Solution LDFCF-15.

(a) $\sigma^2 \approx [1/(n-p)] \sum_{AY,t} (c_{AY,t} - \mu_{AY,t})^2 / \mu_{AY,t}$.

(b) This expression is equivalent to a **chi-square error term**.

(c) It is assumed that the actual incremental loss emergence c follows an **over-dispersed Poisson** distribution. (Clark, p. 9)

Problem LDFCF-16. You are given that, for a *standard* Poisson distribution,

$$\Pr(x) = \lambda^x \exp(-\lambda) / x! \text{ and } E[x] = \text{Var}(x) = \lambda.$$

Now suppose that you are working with an *over-dispersed* Poisson distribution for actual loss amount $c = x \cdot \sigma^2$. The parameter of this over-dispersed Poisson distribution is also λ .

(a) What is the expression for $\Pr(c)$, the probability of any given loss amount c ?

(b) What is the expression for $E[c] = \mu$, the expected value of c ?

(c) What is the expression for $\text{Var}(c)$, the variance of c , in terms of λ ?

(d) What is the expression for $\text{Var}(c)$, the variance of c , in terms of μ ?

(Clark, p. 10)

Solution LDFCF-16.

(a) $\Pr(c) = \lambda^{c/\sigma^2} \exp(-\lambda) / (c/\sigma^2)!$

(b) $E[c] = \mu = \lambda \cdot \sigma^2$.

(c) $\text{Var}(c) = \lambda \cdot \sigma^4$.

(d) $\text{Var}(c) = \mu \cdot \sigma^2$.

Problem LDFCF-17. What are two advantages of using the over-dispersed Poisson model for actual loss amounts? (Clark, p. 10)

Solution LDFCF-17.

Advantage 1. The scaling factor in the model allows matching of the first and second moments of any distribution, which gives the model a high degree of flexibility.

Advantage 2. Maximum likelihood estimation exactly produces the LDF and Cape Cod estimates of ultimate. These results can be presented in a format familiar to reserving actuaries. (Clark, p. 10)

Problem LDFCF-18.

(a) Why, according to Clark, is it not a concern that, using an over-dispersed Poisson model, the distribution of ultimate reserves is approximated by a discretized (rather than a continuous) curve?

(b) What advantage does the use of a discrete distribution have in the context of modeling loss emergence? (Clark, p. 10)

Solution LDFCF-18.

(a) The scale factor σ^2 is usually small relative to the mean, so little precision is lost from using a discretized curve.

(b) A discrete distribution allows for a probability mass point at zero, which represents cases where there is no change in loss in a given development increment. (Clark, p. 10)

Problem LDFCF-19. A general expression for the likelihood function of x is $\prod_i [\Pr(x_i)]$.

(a) Given that the actual loss amount $c = x \cdot \sigma^2$ follows an over-dispersed Poisson distribution, with values c_i and parameter λ_i for each i , what is the formula for the likelihood function for c , in terms of c and λ ?

(b) Given that $E[c] = \mu = \lambda \cdot \sigma^2$, what is the formula for the likelihood function for c , in terms of c and μ ? (Clark, p. 10)

(c) What is the formula for the loglikelihood function of c , which is the natural logarithm of the likelihood function?

(d) In this loglikelihood function, if the scale parameter σ^2 is known, then the maximum likelihood estimation becomes a matter of maximizing *which* quantity? (Clark, p. 11)

Solution LDFCF-19. (a) **Likelihood** = $\prod_i [\lambda_i^{c_i/\sigma^2} \exp(-\lambda_i) / (c_i/\sigma^2)!]$.

(b) **Likelihood** = $\prod_i [(\mu_i/\sigma^2)^{c_i/\sigma^2} \exp(-\mu_i/\sigma^2) / (c_i/\sigma^2)!]$.

(c) **LogLikelihood** = $\sum_i [(c_i/\sigma^2) \ln(\mu_i/\sigma^2) - \mu_i/\sigma^2 - \ln((c_i/\sigma^2)!)]$

(d) If σ^2 is known, then the quantity to be maximized becomes $\sum_i [(c_i) \ln(\mu_i) - \mu_i]$.

Problem LDFCF-20. You are given the following notation:

- $c_{i,t}$ = Actual loss in accident year i , development period t
- P_i = Premium for accident year i
- x_{t-1} = Beginning age for development period t
- x_t = Ending age for development period t
- $G(x)$ = Cumulative distribution function of x

You are using Clark's Cape Cod model. Let ELR be the expected loss ratio.

(a) What is the formula for the loglikelihood function?

(b) What is the formula for the first derivative of the loglikelihood function with respect to ELR?

(c) What is the formula for the maximum likelihood estimate (MLE) of ELR?

(Clark, pp. 11-12)

Solution LDFCF-20.

(a) **LogLikelihood** = $\sum_{i,t} (c_{i,t} \ln(\text{ELR} \cdot P_i \cdot [G(x_t) - G(x_{t-1})]) - \text{ELR} \cdot P_i \cdot [G(x_t) - G(x_{t-1})])$.

(b) $\partial(\text{LogLikelihood})/\partial(\text{ELR}) = \sum_{i,t} (c_{i,t}/\text{ELR} - P_i \cdot [G(x_t) - G(x_{t-1})])$.

(c) MLE of ELR: $\text{ELR} = \sum_{i,t} (c_{i,t}) / \sum_{i,t} (P_i \cdot [G(x_t) - G(x_{t-1})])$.

Problem LDFCF-21. You are given the following notation:

- $c_{i,t}$ = Actual loss in accident year i , development period t
- ULT_i = Ultimate loss for accident year i
- x_{t-1} = Beginning age for development period t
- x_t = Ending age for development period t
- $G(x)$ = Cumulative distribution function of x

You are using Clark's LDF model.

(a) What is the formula for the loglikelihood function?

(b) What is the formula for the first derivative of the loglikelihood function with respect to ULT_i ?

(c) What is the formula for the maximum likelihood estimate (MLE) of ULT_i ? (Clark, p. 12)

Solution LDFCF-21.

(a) **LogLikelihood** = $\sum_{i,t} (c_{i,t} * \ln(ULT_i * [G(x_t) - G(x_{t-1})]) - ULT_i * [G(x_t) - G(x_{t-1})])$.

(b) $\partial(\text{LogLikelihood})/\partial(\text{ELR}) = \sum_t (c_{i,t}/ULT_i - [G(x_t) - G(x_{t-1})])$.

(c) MLE of ULT_i : $ULT_i = \sum_t (c_{i,t}) / \sum_t ([G(x_t) - G(x_{t-1})])$.

Problem LDFCF-22. What is an advantage that the maximum likelihood estimates for ELR using the Cape Cod method and each ULT_i using the LDF method would present in terms of overcoming the problem of overparametrization? (Clark, p. 12)

Solution LDFCF-22. Each maximum likelihood estimate can be set based on the parameters θ and ω from the parametric curve, thereby reducing the 3 parameters of the Cape Cod method to 2 and reducing the $(n + 2)$ parameters of the LDF method to 2. (Clark, p. 12)

Problem LDFCF-23. Fill in the blanks (Clark, p. 12): Using Clark's model, the maximum loglikelihood function never takes the logarithm of _____. Therefore, the model will work even if some of these amounts are _____ or _____.

Solution LDFCF-23. Using Clark's model, the maximum loglikelihood function never takes the logarithm of **the actual incremental development**. Therefore, the model will work even if some of these amounts are **zero** or **negative**. (Clark, p. 12)

Problem LDFCF-24. Suppose you are using Mack's Cape Cod method with parameters ELR, ω , and θ . Suppose the loglikelihood function is denoted as $l_{y,t}$ for each accident year y and development period t .

Fill in the values of the 3-by-3 *second-derivative information matrix* $[I]$ below for Mack's Cape Cod method using the following second-derivative notation: ${}_{y,t}\Sigma[(\partial^2 l_{y,t})/\partial \square]$, where \square will vary depending on the matrix entry. (Clark, p. 13)

Solution LDFCF-24. Consider the matrix as having ELR, ω , and θ as horizontal and vertical labels:

	ELR	ω	θ
ELR			
ω			
θ			

Then, for each matrix entry, take the partial derivative of $l_{y,t}$ with respect to the horizontal variable and then the partial derivative of $l_{y,t}$ with respect to the vertical variable (in either order).

The matrix will look as follows.

	ELR	ω	θ
ELR	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial(\text{ELR})^2)]$	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial(\text{ELR})\partial\omega)]$	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial(\text{ELR})\partial\theta)]$
ω	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\omega\partial(\text{ELR}))]$	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\omega^2)]$	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\omega\partial\theta)]$
θ	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\theta\partial(\text{ELR}))]$	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\theta\partial\omega)]$	$y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\theta^2)]$

Problem LDFCF-25. Suppose you are using Mack's Cape Cod method with parameters ELR, ω , and θ . Let σ^2 be the (Variance/Mean) scaling factor. Let $[I]$ be the second-derivative information matrix. Let $[\Sigma]$ be the covariance matrix.

- (a) Provide an inequality expressing $[\Sigma]$ as being greater than or equal to a matrix expressed in terms of $[I]$.
 (b) Write out the terms of the 3-by-3 matrix $[\Sigma]$. (Clark, p. 13)

Solution LDFCF-25.

- (a) $[\Sigma] \geq -\sigma^2 * [I]^{-1}$, where $[I]^{-1}$ is the inverse of $[I]$.
 (b) The terms of $[\Sigma]$ are the following:

Var(ELR)	Cov(ELR, ω)	Cov(ELR, θ)
Cov(ω , ELR)	Var(ω)	Cov(ω , θ)
Cov(θ , ELR)	Cov(θ , ω)	Var(θ)

Problem LDFCF-26. Suppose you are using Mack's LDF method with four accident years, and separate ultimate-loss estimates for each accident year. The parameters are ULT_1 , ULT_2 , ULT_3 , ULT_4 , ω , and θ . Suppose the loglikelihood function is denoted as $l_{y,t}$ for each accident year y and development period t .

Fill in the values of the 6-by-6 *second-derivative information matrix* $[I]$ for Mack's LDF method using the following second-derivative notation: $y_{y,t}\Sigma[(\partial^2 l_{y,t})/(\partial\Box)]$, where \Box will vary depending on the matrix entry, and the value of y may also vary by accident year. (Clark, p. 14)

Solution LDFCF-26.

Consider the matrix as having ULT_1 , ULT_2 , ULT_3 , ULT_4 , ω , and θ as horizontal and vertical labels.

Then, for each matrix entry, take the partial derivative of $l_{y,t}$ with respect to the horizontal variable and then the partial derivative of $l_{y,t}$ with respect to the vertical variable (in either order).

When a partial derivative is taken with respect to one ULT parameter and then with respect to a different ULT parameter, the result will be 0, since the ultimate losses for different accident years are independent of one another. For any derivative with respect to ULT_n (where n is a specific accident year) the summation will only be taken over t , and will be a partial derivative of $l_{n,t}$ for just that n , rather than over all y .

The matrix will look as follows.

	ULT_1	ULT_2	ULT_3	ULT_4	ω	θ
ULT_1	$\frac{\sum[(\partial^2 l_{1,t})/\partial(ULT_1)^2]}{}$	0	0	0	$\frac{\sum[(\partial^2 l_{1,t})/\partial(ULT_1)\partial(\omega)]}{}$	$\frac{\sum[(\partial^2 l_{1,t})/\partial(ULT_1)\partial(\theta)]}{}$
ULT_2	0	$\frac{\sum[(\partial^2 l_{2,t})/\partial(ULT_2)^2]}{}$	0	0	$\frac{\sum[(\partial^2 l_{2,t})/\partial(ULT_2)\partial(\omega)]}{}$	$\frac{\sum[(\partial^2 l_{2,t})/\partial(ULT_2)\partial(\theta)]}{}$
ULT_3	0	0	$\frac{\sum[(\partial^2 l_{3,t})/\partial(ULT_3)^2]}{}$	0	$\frac{\sum[(\partial^2 l_{3,t})/\partial(ULT_3)\partial(\omega)]}{}$	$\frac{\sum[(\partial^2 l_{3,t})/\partial(ULT_3)\partial(\theta)]}{}$
ULT_4	0	0	0	$\frac{\sum[(\partial^2 l_{4,t})/\partial(ULT_4)^2]}{}$	$\frac{\sum[(\partial^2 l_{4,t})/\partial(ULT_4)\partial(\omega)]}{}$	$\frac{\sum[(\partial^2 l_{4,t})/\partial(ULT_4)\partial(\theta)]}{}$
ω	$\frac{\sum[(\partial^2 l_{1,t})/\partial(\omega)\partial(ULT_1)]}{}$	$\frac{\sum[(\partial^2 l_{2,t})/\partial(\omega)\partial(ULT_2)]}{}$	$\frac{\sum[(\partial^2 l_{3,t})/\partial(\omega)\partial(ULT_3)]}{}$	$\frac{\sum[(\partial^2 l_{4,t})/\partial(\omega)\partial(ULT_4)]}{}$	$\sum_{y,t}[(\partial^2 l_{y,t})/\partial\omega^2]$	$\sum_{y,t}[(\partial^2 l_{y,t})/(\partial\omega\partial\theta)]$
θ	$\frac{\sum[(\partial^2 l_{1,t})/\partial(\theta)\partial(ULT_1)]}{}$	$\frac{\sum[(\partial^2 l_{2,t})/\partial(\theta)\partial(ULT_2)]}{}$	$\frac{\sum[(\partial^2 l_{3,t})/\partial(\theta)\partial(ULT_3)]}{}$	$\frac{\sum[(\partial^2 l_{4,t})/\partial(\theta)\partial(ULT_4)]}{}$	$\sum_{y,t}[(\partial^2 l_{y,t})/(\partial\theta\partial\omega)]$	$\sum_{y,t}[(\partial^2 l_{y,t})/\partial\theta^2]$

Problem LDFCF-27. You are given an estimate of loss reserves R . Let $\mu_{AY:x,y}$ be the expected incremental loss amount in accident year AY between ages x and y . Let $\Sigma(\mu_{AY:x,y})$ be the sum of such incremental loss amounts over a group of periods. Let σ^2 be (Variance/Mean) scale factor. Furthermore, let $[\Sigma]$ be the covariance matrix of parameters and let $[\partial R]$ be the vector of partial derivatives of R with respect to each parameter of the method being used (either Clark's Cape Cod method or Clark's LDF method). Let $[\partial R]^T$ be the transpose of vector $[\partial R]$.

- Provide an expression for the *process variance* of R .
- Provide an expression for the *parameter variance / estimation error* of R .
- If Clark's Cape Cod method with parameters ELR , ω , and θ is used, state the vector ∂R .
- If Clark's LDF method with parameters ULT_1 , ULT_2 , ULT_3 , ω , and θ is used, state the vector ∂R . (Clark, p. 14)

Solution LDFCF-27. (a) Process Variance of R : $\sigma^2 * \Sigma(\mu_{AY:x,y})$

(b) Parameter Variance of R : $[\partial R]^T * [\Sigma] * [\partial R]$

(c) Cape Cod Method: $[\partial R] = \langle \partial R / \partial(ELR), \partial R / \partial\omega, \partial R / \partial\theta \rangle$

(d) LDF Method: $[\partial R] = \langle \partial R / \partial(ULT_1), \partial R / \partial(ULT_2), \partial R / \partial(ULT_3), \partial R / \partial\omega, \partial R / \partial\theta \rangle$

Problem LDFCF-28. Using Clark's Cape Cod method with parameters ELR , ω , and θ , suppose that P_i is the premium for any accident period i under consideration. It is desired to model loss emergence from time x_i to time y_i in each accident period i . Let $G(t)$ be the cumulative distribution function for the distribution of loss emergence (described by parameters ω and θ).

- What is the formula for the future reserve R ?
- What is the formula for $\partial R / \partial(ELR)$?
- What is the formula for $\partial R / \partial\omega$?

(d) What is the formula for $\partial R / \partial \theta$? (Clark, p. 15)

Solution LDFCF-28.

(a) $R = \sum_i [P_i * ELR * (G(y_i) - G(x_i))]$

(b) $\partial R / \partial (ELR) = \sum_i [P_i * (G(y_i) - G(x_i))]$

(c) $\partial R / \partial \omega = \sum_i [P_i * ELR * (\partial [G(y_i)] / \partial \omega - \partial [G(x_i)] / \partial \omega)]$

(d) $\partial R / \partial \theta = \sum_i [P_i * ELR * (\partial [G(y_i)] / \partial \theta - \partial [G(x_i)] / \partial \theta)]$

Problem LDFCF-29. You are using Clark's LDF method with parameters ULT_i , ω , and θ . It is desired to model loss emergence from time x_i to time y_i in each accident period i . Let $G(t)$ be the cumulative distribution function for the distribution of loss emergence (described by parameters ω and θ).

(a) What is the formula for the future reserve R ?

(b) What is the formula for $\partial R / \partial (ULT_2)$?

(c) What is the formula for $\partial R / \partial \omega$?

(d) What is the formula for $\partial R / \partial \theta$?

(Clark, p. 15)

Solution LDFCF-29.

(a) $R = \sum_i [ULT_i (G(y_i) - G(x_i))]$

(b) $\partial R / \partial (ULT_2) = (G(y_2) - G(x_2))$ (For $i \neq 2$, all the other terms are constant with respect to ULT_2 , and so their partial derivative with respect to ULT_2 is 0.)

(c) $\partial R / \partial \omega = \sum_i [ULT_i * (\partial [G(y_i)] / \partial \omega - \partial [G(x_i)] / \partial \omega)]$

(d) $\partial R / \partial \theta = \sum_i [ULT_i * (\partial [G(y_i)] / \partial \theta - \partial [G(x_i)] / \partial \theta)]$

Problem LDFCF-30. Identify the three key assumptions of Clark's model. (Clark, pp. 16-17)

Solution LDFCF-30.

Assumption 1. Incremental losses are independent and identically distributed (iid).

Assumption 2. The Variance/Mean Scale Parameter σ^2 is fixed and known.

Assumption 3. Variance estimates are based on an approximation to the Rao-Cramer lower bound. (Clark, pp. 16-17)

Problem LDFCF-31.

(a) What does the *independence* assumption of incremental losses mean in a reserving context?

(b) Give an example of how this assumption might be violated in reality and a *positive* correlation might be present instead.

(c) Give an example of how this assumption might be violated in reality and a *negative* correlation might be present instead. (Clark, p. 16)

Solution LDFCF-31.

(a) The independence assumption means that one period does not affect the surrounding periods.

(b) A positive correlation might exist if all periods are equally affected by an increase in loss inflation.

(c) A negative correlation might exist if a large settlement in one period replaces a stream of payments in later periods. (Clark, p. 16)

Problem LDFCF-32.

- (a) What does the “identically distributed” assumption of incremental losses mean in a reserving context?
- (b) What are two reasons why this assumption is unrealistic? (Clark, p. 16)

Solution LDFCF-32.

- (a) The “identically distributed” assumption means that the emergence pattern is the same for all accident years.
- (b) This assumption is unrealistic because:
1. Different historical periods would have had different risks and a different mix of business written in each; and
 2. Different historical periods would have been subject to different claim-handling and settlement strategies. (Clark, p. 16)

Problem LDFCF-33. Fill in the blanks (Clark, p. 17): Via the assumption that the Variance/Mean Scale Parameter σ^2 is fixed and known, one is essentially ignoring the variance on the _____. In classical statistics, this assumption is typically relaxed by using the _____ distribution instead of the Normal distribution. If this assumption is relaxed in a reserving context, the reserve ranges would _____ [increase, decrease, or stay the same?].

Solution LDFCF-33. Via the assumption that the Variance/Mean Scale Parameter σ^2 is fixed and known, one is essentially ignoring the variance on the **variance**. In classical statistics, this assumption is typically relaxed by using the **Student-T** distribution instead of the Normal distribution. If this assumption is relaxed in a reserving context, the reserve ranges would **increase**. (Clark, p. 17)

Problem LDFCF-34.

- (a) Why is it necessary to use a Rao-Cramer lower bound as the variance estimate in Clark’s model? (Clark, p. 17)
- (b) **Fill in the blanks** (Clark, p. 17): Technically, the Rao-Cramer lower bound is based on the true expected values of the _____. However, because true _____ are not known, estimated values must be utilized, and the result is called the _____ information matrix, rather than the _____ information matrix.

Solution LDFCF-34.

- (a) Only linear functions are amenable to exact estimates of variance based on the information matrix. Clark’s model utilizes non-linear functions and so requires the use of the Rao-Cramer lower bound as an approximation. (Clark, p. 17)
- (b) Technically, the Rao-Cramer lower bound is based on the true expected values of the **second-derivative matrix**. However, because true **parameters** are not known, estimated values must be utilized, and the result is called the **observed** information matrix, rather than the **expected** information matrix. (Clark, p. 17)

Problem LDFCF-35. You are given the following *cumulative* loss-development triangle for paid claims:

	12 months	24 months	36 months	48 months	60 months	72 months
AY 2333	352,352	644,642	689,432	722,367	800,900	811,001
AY 2334	352,123	555,424	712,900	815,515	818,923	
AY 2335	403,356	666,666	803,535	825,567		
AY 2336	399,218	544,554	600,666			
AY 2337	444,444	612,336				
AY 2338	422,198					

(a) Create the corresponding *incremental* loss-development triangle.

(b) Rearrange the values of the incremental loss-development triangle into the tabular format presented by Clark (pp. 19, 36).

(c) Now suppose that only the latest three maturities are available for each accident year as follows:

	12 months	24 months	36 months	48 months	60 months	72 months
AY 2333				722,367	800,900	811,001
AY 2334			712,900	815,515	818,923	
AY 2335		666,666	803,535	825,567		
AY 2336	399,218	544,554	600,666			
AY 2337	444,444	612,336				
AY 2338	422,198					

Create the corresponding *incremental* loss-development triangle.

(d) Rearrange this partial incremental loss-development triangle into the tabular format presented by Clark (pp. 20, 37).

Solution LDFCF-35.

(a) The *incremental* loss-development triangle looks as follows:

	12 months	24 months	36 months	48 months	60 months	72 months
AY 2333	352,352	292,290	44,790	32,935	78,533	10,101
AY 2334	352,123	203,301	157,476	102,615	3,408	
AY 2335	403,356	263,310	136,869	22,032		
AY 2336	399,218	145,336	56,112			
AY 2337	444,444	167,892				
AY 2338	422,198					

(b) The tabular format of the incremental triangle is as follows:

Accident Year	From	To	Increment	Diagonal Age	Total AY Loss
2333	0 months	12 months	352,352	72 months	811,001
2333	12 months	24 months	292,290	72 months	811,001
2333	24 months	36 months	44,790	72 months	811,001
2333	36 months	48 months	32,935	72 months	811,001
2333	48 months	60 months	78,533	72 months	811,001
2333	60 months	72 months	10,101	72 months	811,001
2334	0 months	12 months	352,123	60 months	818,923
2334	12 months	24 months	203,301	60 months	818,923
2334	24 months	36 months	157,476	60 months	818,923
2334	36 months	48 months	102,615	60 months	818,923
2334	48 months	60 months	3,408	60 months	818,923
2335	0 months	12 months	403,356	48 months	825,567
2335	12 months	24 months	263,310	48 months	825,567
2335	24 months	36 months	136,869	48 months	825,567
2335	36 months	48 months	22,032	48 months	825,567
2336	0 months	12 months	399,218	36 months	600,666
2336	12 months	24 months	145,336	36 months	600,666
2336	24 months	36 months	56,112	36 months	600,666
2337	0 months	12 months	444,444	24 months	612,336
2337	12 months	24 months	167,892	24 months	612,336
2338	0 months	12 months	422,198	12 months	422,198

(c) The partial *incremental* loss-development triangle looks as follows:

	12 months	24 months	36 months	48 months	60 months	72 months
AY 2333				722,367	78,533	10,101
AY 2334			712,900	102,615	3,408	
AY 2335		666,666	136,869	22,032		
AY 2336	399,218	145,336	56,112			
AY 2337	444,444	167,892				
AY 2338	422,198					

(b) The tabular format of the partial incremental triangle is as follows:

Accident Year	From	To	Increment	Diagonal Age	Total AY Loss
2333	0 months	48 months	722,367	72 months	811,001
2333	48 months	60 months	78,533	72 months	811,001
2333	60 months	72 months	10,101	72 months	811,001
2334	0 months	36 months	712,900	60 months	818,923
2334	36 months	48 months	102,615	60 months	818,923
2334	48 months	60 months	3,408	60 months	818,923
2335	0 months	24 months	666,666	48 months	825,567
2335	24 months	36 months	136,869	48 months	825,567
2335	36 months	48 months	22,032	48 months	825,567
2336	0 months	12 months	399,218	36 months	600,666
2336	12 months	24 months	145,336	36 months	600,666
2336	24 months	36 months	56,112	36 months	600,666
2337	0 months	12 months	444,444	24 months	612,336
2337	12 months	24 months	167,892	24 months	612,336
2338	0 months	12 months	422,198	12 months	422,198

Problem LDFCF-36. According to Clark, what is a common practical difficulty with development triangles that the use of the tabular format can easily resolve? (Clark, pp. 20-21)

Solution LDFCF-36. A common practical difficulty is the use of **irregular evaluation periods** (other than multiples of 12 months or another recurring accident period). If the tabular format, this can be accommodated by simply changing the fields pertaining to evaluation age – e.g., the “From”, “To”, and “Diagonal Age” fields. (Clark, pp. 20-21)

Problem LDFCF-37. Once data from a loss-development triangle has been arranged in a tabular format, a parametric curve can be fitted to the data. How are the fitted parameters typically found? (Clark, p. 21)

Solution LDFCF-37. The fitted parameters are typically found via **iteration**, using a statistical software package or a spreadsheet.

Problem LDFCF-38. You are given the following in Clark’s model:

- σ^2 is the (Variance/Mean) scale parameter.
- $\mu^{\wedge}_{AY:x,y}$ is the estimated expected loss emergence for accident year AY from time x to time y.
- $c_{AY:x,y}$ is the actual loss emergence for accident year AY from time x to time y.

Provide the formula for the normalized residual, $r_{AY:x,y}$. (Clark, p. 22)

Solution LDFCF-38. $r_{AY:x,y} = (c_{AY:x,y} - \mu^{\wedge}_{AY:x,y}) / \sqrt{(\sigma^2 * \mu^{\wedge}_{AY:x,y})}$

Problem LDFCF-39. When applying Clark’s model and plotting normalized residuals on the vertical axis against the increment of loss emergence (i.e., increment age) on the horizontal axis, what is the desired outcome of the plot? (Clark, p. 22)

Solution LDFCF-39. The desired outcome of the plot is that the residuals would be randomly scattered around the horizontal zero line, and there would be roughly constant variability across the increment ages. (Clark, p. 22)

Problem LDFCF-40. Clark (pp. 22-23) discusses a plot of normalized residuals on the vertical axis against the expected incremental loss amount on the horizontal axis.

(a) This graph is a useful check on *what* assumption?

(b) What would we observe with regard to the residuals if the assumption in part (a) does not hold?

Solution LDFCF-40.

(a) This graph is a useful check on the assumption that **the variance/mean ratio is constant** (i.e., that the use of a single scale parameter σ^2 is justified).

(b) If the assumption of a constant variance/mean ratio does not hold, then we would expect to see **residuals significantly closer to the zero line at one end of the graph.**

Problem LDFCF-41.

(a) When applying Clark’s model and plotting normalized residuals on the vertical axis, against what two other quantities – besides increment age and expected incremental loss – could normalized residuals be plotted?

(b) What is a desired attribute of each of the plots of normalized residuals in part (a)? (Clark, p. 23)

Solution LDFCF-41.

(a) The normalized residuals could also be plotted against (i) **calendar year of emergence**, or (ii) **accident year.**

(b) The desired attribute of the plots is always that the **residuals appear to be scattered randomly around the zero line** – or else some of the model’s assumptions would be incorrect. (Clark, p. 23)

Problem LDFCF-42.

(a) When loss emergence is fitted to a Loglogistic curve, what does Clark (p. 25) recommend as an effective way of reducing the reliance on extrapolation?

(b) What distribution provides a lighter-tailed alternative to the Loglogistic curve?

Solution LDFCF-42.

(a) Selection of a **truncation point** is an effective way of reducing the reliance on extrapolation.

(b) The **Weibull distribution** provides a lighter-tailed alternative to the Loglogistic curve. (Clark, p. 25)

Problem LDFCF-43.

(a) Using Clark's LDF method, how many observations are there, given a filled-out incremental loss-development triangle with n accident years?

(b) How many parameters are there?

(c) Which is more significant, the process variance or the parameter variance?

(d) What is the main reason for the relationship in part (c) above?

(Clark, p. 25)

Solution LDFCF-43.

(a) There are $n(n+1)/2 = (n^2+n)/2$ observations. (This is the general formula for the sum of consecutive integers from 1 to n , which is applicable here, since there are n rows, starting with n observations from the earliest accident year and decreasing to one observation for the latest accident year.)

(b) There are $(n+2)$ parameters for Clark's LDF method.

(c) The **parameter variance** is more significant.

(d) This is because of **overparametrization**. The $(n^2+n)/2$ observations are not sufficient to estimate the $(n+2)$ parameters.

(Clark, p. 25)

Problem LDFCF-44. According to Clark (p. 26), what is a fundamentally flawed assumption in the LDF method that results in overparametrization?

Solution LDFCF-44. The flawed assumption is that the ultimate loss for each accident year is estimated independently from the ultimate losses in the other accident years – in effect assuming that knowledge of ultimate losses in one year would provide no information about ultimate losses in a proximate years. (Clark, p. 26)

Problem LDFCF-45. To model loss emergence, you are using a Loglogistic curve (with time x being measured in months), with parameters $\omega = 1.6$ and $\theta = 60$. You are given the following information regarding losses emerged to date.

Accident Year	Reported Losses	Age (Months) at 12/31/2048	Average Age (x)	Growth Function	Fitted LDF	Ultimate Losses	Estimated Reserves
2044	5,361,236	60					
2045	3,636,951	48					
2046	3,290,333	36					
2047	3,333,331	24					
2048	1,204,564	12					
TOTAL	16,826,415	-----	-----	-----	-----		

(a) Fill out the rest of the table using Clark's LDF method. (Clark, p. 23)

(b) Now suppose that the Loglogistic curve is to be truncated at 240 months. Fill out the table with the "Fitted LDF" values replaced by a "Truncated LDF", with ultimate losses and reserves adjusted accordingly. (Clark, p. 24)

Solution LDFCF-45.

(a) First, we consider the average age of each year's reported losses. Clark assumes that the average age is the midpoint of the year, and we can reflect this assumption by setting x equal to (Age at 12/31/2048 – 6 months).

Next, we apply the Loglogistic growth curve $G(x \mid \omega, \theta) = x^\omega / (x^\omega + \theta^\omega)$. Therefore, for each value of Average Age (x), the corresponding value of the curve will be $G(x \mid 1.6, 60) = x^{1.6} / (x^{1.6} + 60^{1.6})$.

The Fitted LDF is equal to $1 / (\text{Growth Function})$.

The Ultimate Losses are equal to (Reported Losses) * (Fitted LDF).

The Estimated Reserves are equal to (Ultimate Losses) – (Reported Losses).

The filled-out table looks as follows:

Accident Year	Reported Losses	Age (Months) at 12/31/2048	Average Age (x)	Growth Function	Fitted LDF	Ultimate Losses	Estimated Reserves
2044	5,361,236	60	54	0.45795532	2.18361915	11,706,898	6,345,662
2045	3,636,951	48	42	0.36107995	2.76946972	10,072,426	6,435,475
2046	3,290,333	36	30	0.24805075	4.03143313	13,264,757	9,974,424
2047	3,333,331	24	18	0.12715441	7.8644539	26,214,828	22,881,497
2048	1,204,564	12	6	0.02450337	40.8107171	49,159,121	47,954,557
TOTAL	16,826,415	-----	-----	-----	-----	110,418,029	93,591,614

(b) Now applying the truncation factor at 240 months, this gives us an average age at truncation of $240 - 6 = 234$ months. $G(234 \mid 1.6, 60) = 234^{1.6} / (234^{1.6} + 60^{1.6}) = 0.8982164418$.

Thus, the Truncated LDF is equal to $0.8982164418 / (\text{Growth Function})$.

The rest of the calculations follow the same format as those in part (a) above.

The Ultimate Losses are equal to (Reported Losses) * (Truncated LDF).

The Estimated Reserves are equal to (Ultimate Losses) – (Reported Losses).

Accident Year	Reported Losses	Age (Months) at 12/31/2048	Average Age (x)	Growth Function	Truncated LDF	Ultimate Losses	Estimated Reserves
2044	5,361,236	60	54	0.45795532	1.96136263	10,515,328	5,154,092
2045	3,636,951	48	42	0.36107995	2.48758324	9,047,218	5,410,267
2046	3,290,333	36	30	0.24805075	3.62109952	11,914,623	8,624,290
2047	3,333,331	24	18	0.12715441	7.06398179	23,546,589	20,213,258
2048	1,204,564	12	6	0.02450337	36.6568571	44,155,530	42,950,966
TOTAL	16,826,415	-----	-----	-----	-----	99,179,289	82,352,874

Problem LDFCF-46.

(a) To apply the Cape Cod method, with what should the loss-development triangle be supplemented?

(b) What data element is a good supplement for this purpose?

(c) If a further refinement is desired, what additional adjustment could be made?

(Clark, p. 26)

Solution LDFCF-46.

(a) The loss-development triangle should be supplemented with an **exposure base** that is believed to be proportional to ultimate expected losses by accident year.

(b) **On-level premium** – i.e., premium adjusted to a common rate level per exposure – is a good candidate for the exposure base.

(c) An additional adjustment for **loss trend net of exposure trend** could be made so that the cost level is the same for all years, along with the rate level. (Clark, p. 26)

Problem LDFCF-47. Which method – the LDF method or the Cape Cod method – results in a lower overall reserve variance and standard deviation, and why? (Clark, p. 29)

Solution LDFCF-47. The **Cape Cod method** results in a lower overall reserve variance and standard deviation, because it makes use of more information – e.g., the on-level premium by year – which is not available in the LDF method. The additional information allows a significantly better estimate of the reserve. (Clark, p. 29)

Problem LDFCF-48. To model loss emergence, you are using a Weibull curve (with time x being measured in months), with parameters $\omega = 3$ and $\theta = 40$. You are given the following information regarding on-level premiums and losses emerged to date.

AY	On-Level Premium	Reported Losses	Age (Months) at Dec. 31, 2048	Avg. Age (x)	Growth Function	Premium * Growth Function	Ultimate Loss Ratio	Estimated Reserves
2044	6,666,666	5,361,236	60					
2045	6,800,000	3,636,951	48					
2046	7,888,999	3,290,333	36					
2047	8,200,000	3,333,331	24					
2048	9,120,000	1,204,564	12					
SUM	38,675,665	16,826,415	-----	-----	-----			

Fill out the rest of the table using Clark's Cape Cod method. (Clark, pp. 27-28)

Solution LDFCF-48.

First, we consider the average age of each year's reported losses. Clark assumes that the average age is the midpoint of the year, and we can reflect this assumption by setting x equal to (Age at 12/31/2048 – 6 months).

Next, we apply the Weibull growth curve $G(x \mid \omega, \theta) = 1 - \exp(-(x/\theta)^\omega)$. Therefore, for each value of Average Age (x), the corresponding value of the curve will be $G(x \mid 3, 40) = 1 - \exp(-(x/40)^3)$. These will be the values of the "Growth Function" column.

Next, we multiply the values in the “On-Level Premium” column by the corresponding values in the “Growth Function” column to get the values in the “Premium * Growth Function” column.

The values in the “Ultimate Loss Ratio” column are equal to (Reported Losses)/(Premium * Growth Function). The total Ultimate Loss Ratio (the Cape Cod ELR) is equal to $16,826,415/14,220,724 = 1.183232021 = 118.3232021\%$.

Estimated Reserves for each row are equal to (On-Level Premium)*(Cape Cod ELR)*(1-Growth Function) = $118.3232021\% * (\text{On-Level Premium}) * (1 - \text{Growth Function})$. The total estimated reserve is thus 28,935,867.

The filled-out table looks as follows:

AY	On-Level Premium	Reported Losses	Age (Months) at Dec. 31, 2048	Avg. Age (x)	Growth Function	Premium * Growth Function	Ultimate Loss Ratio	Estimated Reserves
2044	6,666,666	5,361,236	60	54	0.9145971	6,097,313	87.93%	673,676
2045	6,800,000	3,636,951	48	42	0.6857684	4,663,225	77.99%	2,528,300
2046	7,888,999	3,290,333	36	30	0.3441840	2,715,267	121.18%	6,121,725
2047	8,200,000	3,333,331	24	18	0.0870964	714,191	466.73%	8,857,449
2048	9,120,000	1,204,564	12	6	0.0033693	30,728	3920.07%	10,754,717
SUM	38,675,665	16,826,415	-----	-----	-----	14,220,724	118.3232%	28,935,867

Problem LDFCF-49. When one uses Clark’s Cape Cod method and graphs ultimate loss ratios by year, what observed pattern would indicate a concern regarding bias introduced into the reserve estimate? (Clark, p. 28)

Solution LDFCF-49. A pattern of **increasing or decreasing ultimate loss ratios** would indicate a concern regarding possible bias. (Clark, p. 28)

Problem LDFCF-50. You are given the following estimated loss emergence and estimated reserves using Clark’s LDF method and a Loglogistic curve (with time x being measured in months), with parameters $\omega = 1.6$ and $\theta = 60$.

Accident Year	Reported Losses	Age (Months) at 12/31/2048	Average Age (x)	Growth Function	Fitted LDF	Ultimate Losses	Estimated Reserves
2044	5,361,236	60	54	0.45795532	2.18361915	11,706,898	6,345,662
2045	3,636,951	48	42	0.36107995	2.76946972	10,072,426	6,435,475
2046	3,290,333	36	30	0.24805075	4.03143313	13,264,757	9,974,424
2047	3,333,331	24	18	0.12715441	7.8644539	26,214,828	22,881,497
2048	1,204,564	12	6	0.02450337	40.8107171	49,159,121	47,954,557
TOTAL	16,826,415	-----	-----	-----	-----	110,418,029	93,591,614

Provide estimates for loss development for each accident year and in total during the next 12 months – i.e., during the period from 1/1/2049 to 12/31/2049. (Clark, p. 31)

Solution LDFCF-50.

First, we consider the average age of each year's reported losses. Clark assumes that the average age is the midpoint of the year. The average age at 12/31/2048 is given, so the average age at 12/31/2049 for each accident year's experience is 12 months greater.

The corresponding Growth Functions at 12/31/2049 for AYs 2045 through 2048 have already been calculated. (They are the same as the growth functions at 12/31/2048 for the immediately preceding accident years.) It remains to apply the Loglogistic growth curve

$$G(x \mid \omega, \theta) = x^\omega / (x^\omega + \theta^\omega) \text{ to calculate } G(66 \mid 1.6, 60) = 66^{1.6} / (66^{1.6} + 60^{1.6}) = 0.53805036.$$

The Estimated 12-Month Development in the table below is equal to (Ultimate Losses)*(Growth Function at 12/31/2049 - Growth Function at 12/31/2048).

Accident Year	Reported Losses	Average Age (x) at 12/31/2048	Average Age (x) at 12/31/2049	Growth Function at 12/31/2048	Growth Function at 12/31/2049	Ultimate Losses	Estimated 12-Month Development
2044	5,361,236	54	66	0.45795532	0.53805036	11,706,898	937,664
2045	3,636,951	42	54	0.36107995	0.45795532	10,072,426	975,770
2046	3,290,333	30	42	0.24805075	0.36107995	13,264,757	1,499,305
2047	3,333,331	18	30	0.12715441	0.24805075	26,214,828	3,169,277
2048	1,204,564	6	18	0.02450337	0.12715441	49,159,121	5,046,235
TOTAL	16,826,415	-----	-----	-----	-----	110,418,029	11,628,251

Problem LDFCF-51. What is an ability that Clark's Cape Cod method provides with regard to forecasting losses for a *prospective* period? How is this made possible? (Clark, p. 30)

Solution LDFCF-51. Clark's Cape Cod method enables forecasting of losses and estimation of reserve variability for a prospective period via the following assumptions:

- The Cape Cod Expected Loss Ratio (ELR) has already been calculated and is assumed to be the same for the prospective period.
 - The on-level earned premium for the prospective period can be estimated via a premium-trend assumption (e.g. a growth rate of $x\%$).
 - The mean expected loss is equal to (On-Level Earned Premium)*(Cape Cod ELR).
 - The Variance/Mean scale parameter can be assumed to be constant, allowing for an estimate of process variance.
 - Parameter variance can be estimated using the covariance matrix for the parameters ELR and ω and θ from the parametric growth curve. These parameters are also unchanged.
 - Total reserve variance is calculated as the sum of process variance and parameter variance.
- (Clark, p. 30)

Problem LDFCF-52.

- (a) According to Clark (p. 32), the mathematics for calculating the variability around discounted reserves follows directly from *which* three elements that are already available from Clark's approach for undiscounted reserves?
- (b) To what kinds of data would the appropriateness of such a calculation be limited?

Solution LDFCF-52. (a) The following already-available elements are necessary:

Element 1. Payout pattern

Element 2. Model parameters

Element 3. Covariance matrix

- (b) Such a calculation would only be appropriate if **paid data** are being analyzed. (E.g., the payout pattern would not suffice if the analysis were made on incurred losses that include case reserves.) (Clark, p. 32)

Problem LDFCF-53.

- (a) Is the coefficient of variation (CV) larger or smaller for a discounted reserve, as compared to an undiscounted reserve?
- (b) What explains the observation in part (a) above? (Clark, p. 32)

Solution LDFCF-53.

- (a) The CV is **smaller** for a discounted reserve.
- (b) The "tail" of the payout curve has the greatest variance (since there is more variability in losses that would be paid a long time from now). With discounting, the tail also receives the deepest discount, as the discount factor is applied to more time periods. (Clark, p. 32)

Problem LDFCF-54.

- (a) Apart from the application of Clark's model, what two elements should be considered in the selection of a reserve range?
- (b) What is a term Clark uses to describe these considerations, and why is this term appropriate? (Clark, p. 34)

Solution LDFCF-54.

- (a) (i) **Changes in mix of business** and (ii) **the process of settling claims** should be considered in the selection of a reserve range.
- (b) These considerations can be described as **model variance**, since they are factors outside of the model's assumptions. (Clark, p. 34)

Problem LDFCF-55. What are three advantages of using the Loglogistic and Weibull parametric curve forms? (Clark, p. 34)

Solution LDFCF-55. The Loglogistic and Weibull parametric curve forms:

1. Smoothly move from 0% to 100%;
2. Often closely match the empirical data;
3. Have directly calculable first and second derivatives, without the need for numerical approximations. (Clark, p. 34)

Problem LDFCF-56. What is generally observed to be greater for reserve estimates, parameter variance or process variance – and why? What would remedy this situation? (Clark, p. 35)

Solution LDFCF-56. Parameter variance is generally observed to be greater than process variance. This indicates that the uncertainty in the estimated reserve is related more to our inability to reliably estimate the expected reserve, rather than to random events. What would remedy this is **more complete data** – e.g., supplementing the loss-development triangle with accident-year exposure information. (Clark, p. 35)